Date: Subject: (S.02)

# الحركة في المستوى

كارتيزية (أ، ب) — قطبية $(\hat{r}, \hat{\theta})$ — طبيعية $(\hat{n}, \hat{t})$

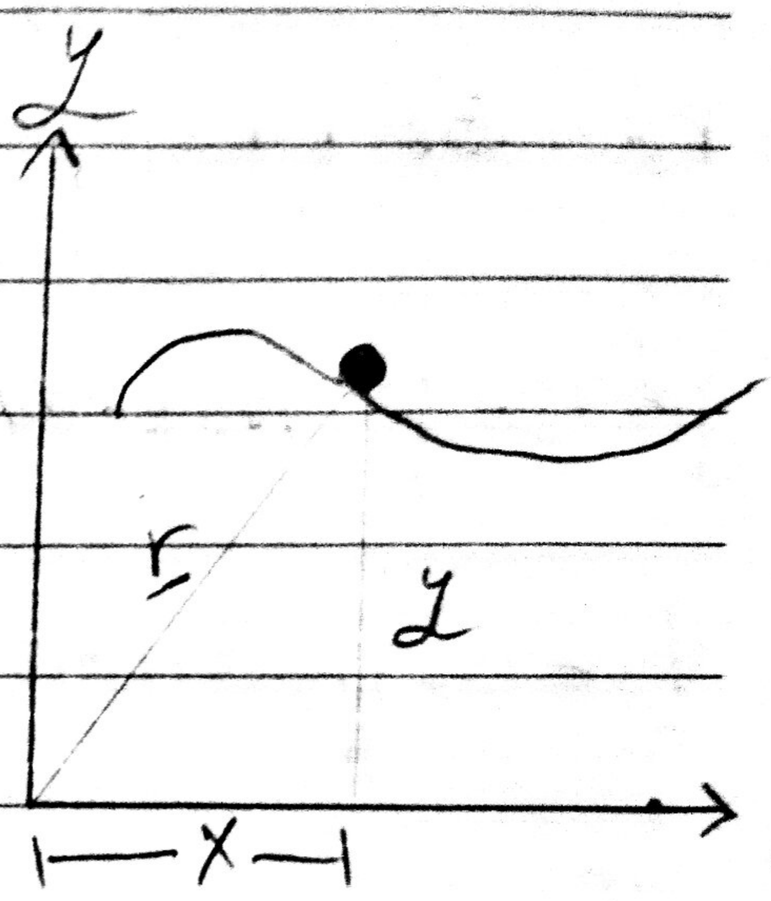
## الإحداثيات الكارتيزية

$$\underline{r} = x\,\underline{i} + y\,\underline{j}$$

$$\underline{V} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\,\underline{i} + \frac{dy}{dt}\,\underline{j}$$

$$\underline{V} = V_x\,\underline{i} + V_y\,\underline{j}$$

($V_x = \dot{x}$, $V_y = \dot{y}$)




$$\underline{F} = \frac{d\underline{V}}{dt} = \frac{dV_x}{dt}\,\underline{i} + \frac{dV_y}{dt}\,\underline{j}$$

$$\underline{F} = F_x\,\underline{i} + F_y\,\underline{j}$$

Ex: $\underline{r} = \underbrace{2t}_{x}\,\underline{i} + \underbrace{3t^2}_{y}\,\underline{j}$

$x = 2t \rightarrow t = \frac{x}{2}$

$y = 3t^2$

$y = 3\left(\frac{x}{2}\right)^2$

Date: Subject:

EX: [9] Pg 17

$\underline{F} = -100\cos 5t\ \underline{i} - 75\sin 5t\ \underline{j}$

$V_0 = 15\underline{J}$

$P_0 = (5, 0)$

Sol

$r_0 = 5\underline{i}$

$\int d\underline{V} = \int (-100\cos 5t\ \underline{i} - 75\sin 5t\ \underline{j})\, dt$

$\underline{V} = -\frac{100}{5}\sin 5t\ \underline{i} + \frac{75}{5}\cos 5t\ \underline{J} + \underline{C}$

$15\underline{J} = 0 + 15\underline{J} + \underline{C} \Rightarrow \underline{C} = 0$

$\underline{V} = -20\sin 5t\ \underline{i} + 15\cos 5t\ \underline{J}$

$\int d\underline{r} = \int (-20\sin 5t\ \underline{i} + 15\cos 5t\ \underline{J})\, dt$

$\underline{r} = \frac{20}{5}\cos 5t\ \underline{i} + \frac{15}{5}\sin 5t\ \underline{J} + \underline{C}$

$5\underline{i} = 4\underline{i} + 0 + \underline{C} \Rightarrow \underline{C} = \underline{i}$

$\underline{r} = [1 + 4\cos 5t]\ \underline{i} + 3\sin 5t\ \underline{J}$

Date: Subject:

$X = 1 + 4\cos 4t$

$y = 3\sin 5t$

$\sin 5t = \frac{y}{3}$

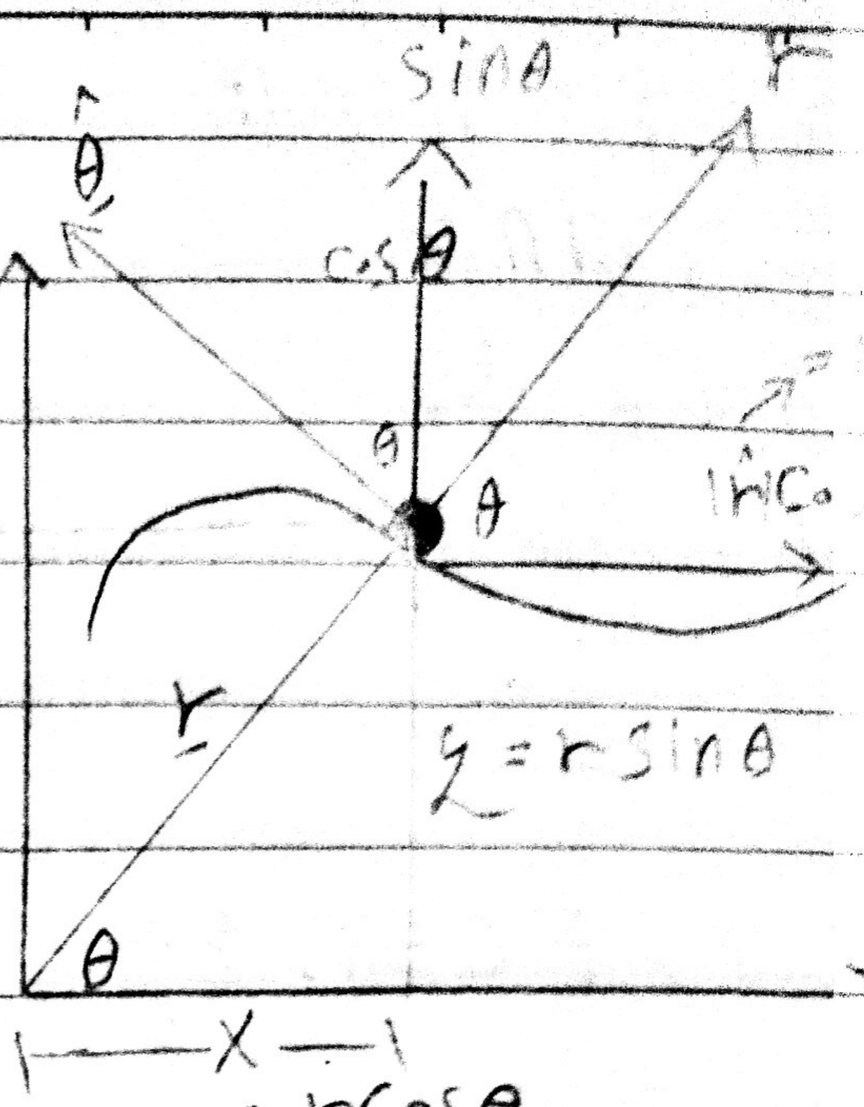
$5t = \sin^{-1}\frac{y}{3}$

$t = \frac{\sin^{-1}\frac{y}{3}}{5}$

$X = 1 + 4\cos\left(4 \cdot \frac{\sin^{-1}\frac{y}{3}}{5}\right)$ //

# الاحداثيات القطبية

V.I.P




$$\underline{r} = X\underline{i} + Y\underline{J}$$

$$= r\cos\theta\,\underline{i} + r\sin\theta\,\underline{J}$$

$$\underline{r} = r\underbrace{[\cos\theta\,\underline{i} + \sin\theta\,\underline{J}]}_{\hat{r}}$$

$$\boxed{\underline{r} = r\hat{r}}$$

$$\hat{A} = \frac{\underline{A}}{A} \Leftarrow \underline{A} = A\hat{A}$$

$$\underline{V} = \frac{dr}{dt} = \frac{d}{dt}(r\hat{r})$$

$$= \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\boxed{\underline{V} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}} \Rightarrow \underline{V} = V_r\hat{r} + V_\theta\hat{\theta}$$

$\dot{\theta}$ ↑ $\dot{\theta}$

$$\frac{d\hat{r}}{dt} = -\sin\theta\cdot\frac{d\theta}{dt}\underline{i} + \cos\theta\cdot\frac{d\theta}{dt}\underline{J}$$

$$= \dot{\theta}\underbrace{[-\sin\theta\,\underline{i} + \cos\theta\,\underline{J}]}_{\hat{\theta}} = \dot{\theta}\hat{\theta}$$

Date: Subject:

$$\underline{f} = \frac{d\underline{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{r}] + \frac{d}{dt}(r\dot{\theta}\hat{\theta})$$

$$= \frac{d\dot{r}}{dt}\hat{r} + \frac{d\hat{r}}{dt}\dot{r} + \frac{dr}{dt}\dot{\theta}\hat{\theta} + \frac{d\dot{\theta}}{dt}r\hat{\theta} + \frac{d\hat{\theta}}{dt}r\dot{\theta}$$

$$= \ddot{r}\hat{r} + \dot{\theta}\hat{\theta}\dot{r} + \dot{r}\dot{\theta}\hat{\theta} + \ddot{\theta}r\hat{\theta} + r\dot{\theta}(-\dot{\theta}\hat{r})$$

$$\frac{d\hat{\theta}}{dt} = -\cos\theta\,\dot{\theta}\,\underline{i} - \sin\theta\,\dot{\theta}\,\underline{j}$$

$$= -\dot{\theta}[\cos\theta\,\underline{i} + \sin\theta\,\underline{j}]$$

$$= -\dot{\theta}\,\hat{r}$$

$$\boxed{\underline{f} = [\ddot{r} - r\dot{\theta}^2]\hat{r} + [2\dot{r}\dot{\theta} + r\ddot{\theta}]\hat{\theta}}$$

$$\underline{f} = f_r\,\hat{r} + f_\theta\,\hat{\theta}$$

Date: Subject:

EX(8)

pg 31.1-

$\theta = t^3$ $r = 100t^2$

V, f $\rightarrow t = 1$ Sec

Sol

$\theta = t^3 = 1$ $r = 100t^2 = 100$

$\dot{\theta} = 3t^2 = 3$ $\dot{r} = 200t = 200$

$\ddot{\theta} = 6t = 6$ $\ddot{r} = 200 = 200$

$V_r = \dot{r} = 200$

$V_\theta = r\dot{\theta} = 300$

$V = \sqrt{V_r^2 + V_\theta^2}$

$f_r = \ddot{r} - r\dot{\theta}^2$

$f_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

$F = \sqrt{f_r^2 + f_\theta^2}$